

24/11/2015

الشرقا

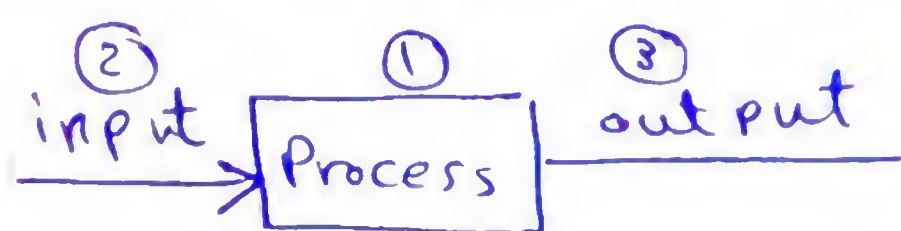
م. شهاب

محاضرة [8]

Control Systems Design :-

Engineering Problems

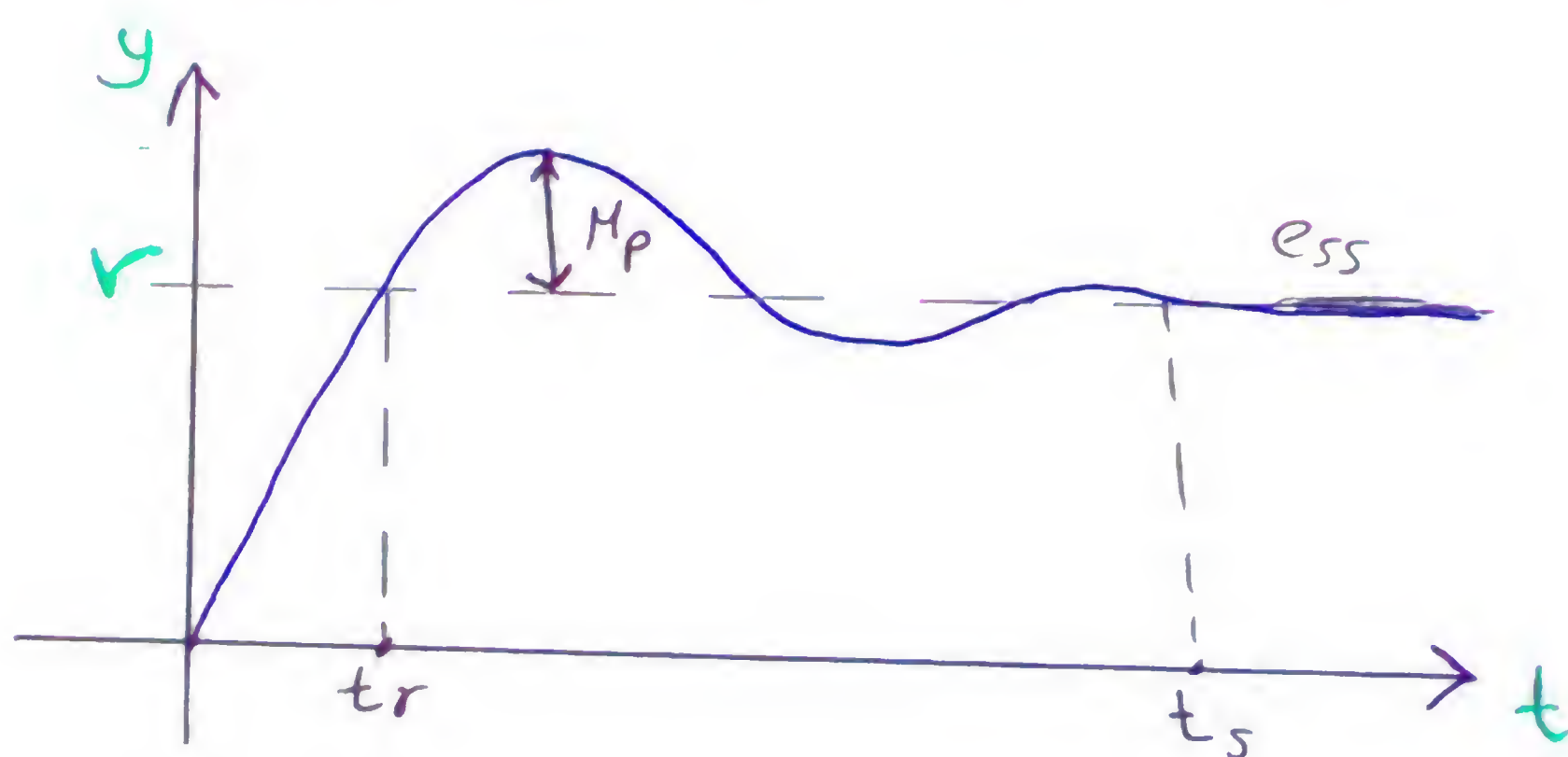
- Analysis Problem:



- Design Problem:



Control Systems Design Specs :-



$$t_r = \frac{\pi - \cos^{-1} \zeta}{\omega_n \sqrt{1 - \zeta^2}} ; \quad t_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}}$$

$$M_p = e^{-\zeta \pi / \sqrt{1 - \zeta^2}} ; \quad t_s = \frac{4}{\zeta \omega_n}$$

e_{ss} :

$$\text{type 0: } e_{ss} = \frac{1}{1 + K_p} ; \quad \text{type 1: } e_{ss} = \frac{1}{K_v}$$

$$\text{type 2: } e_{ss} = \frac{1}{K_a}$$

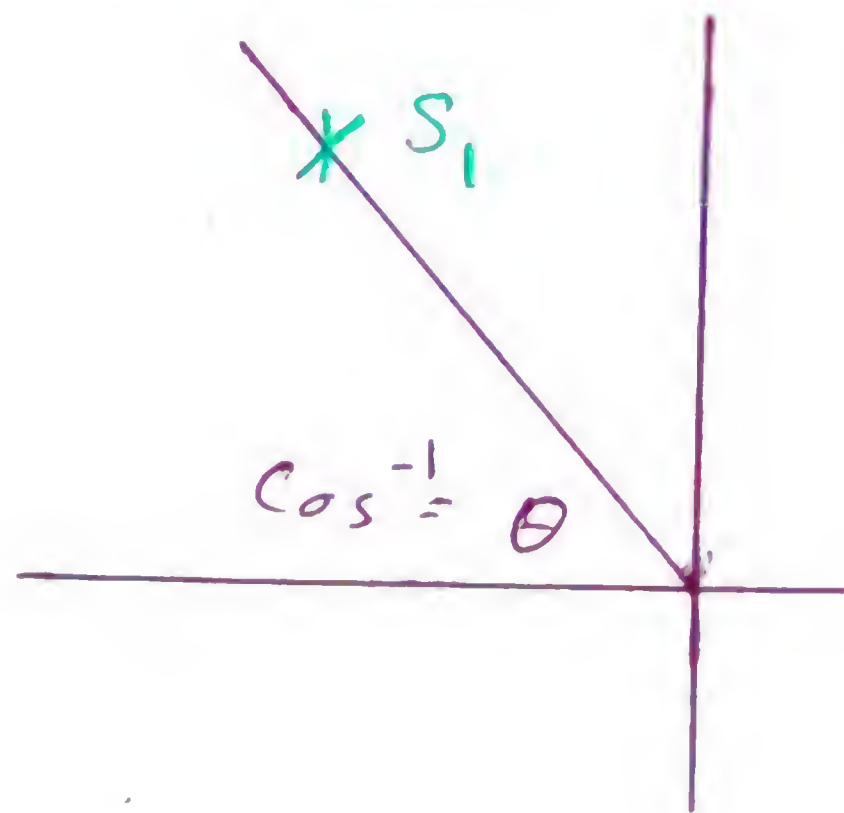
$$K_p = \lim_{s \rightarrow 0} GH(s) ; K_v = \lim_{s \rightarrow 0} s GH(s)$$

$$K_a = \lim_{s \rightarrow 0} s^2 GH(s)$$

* Design Procedure

① desired C.L. c/c eqn

$$s_{1,2} = \underbrace{-\zeta \omega_n}_\alpha \pm \underbrace{\omega_n \sqrt{1 - \zeta^2}}_{\omega_d}$$



- Control Design Objective :-

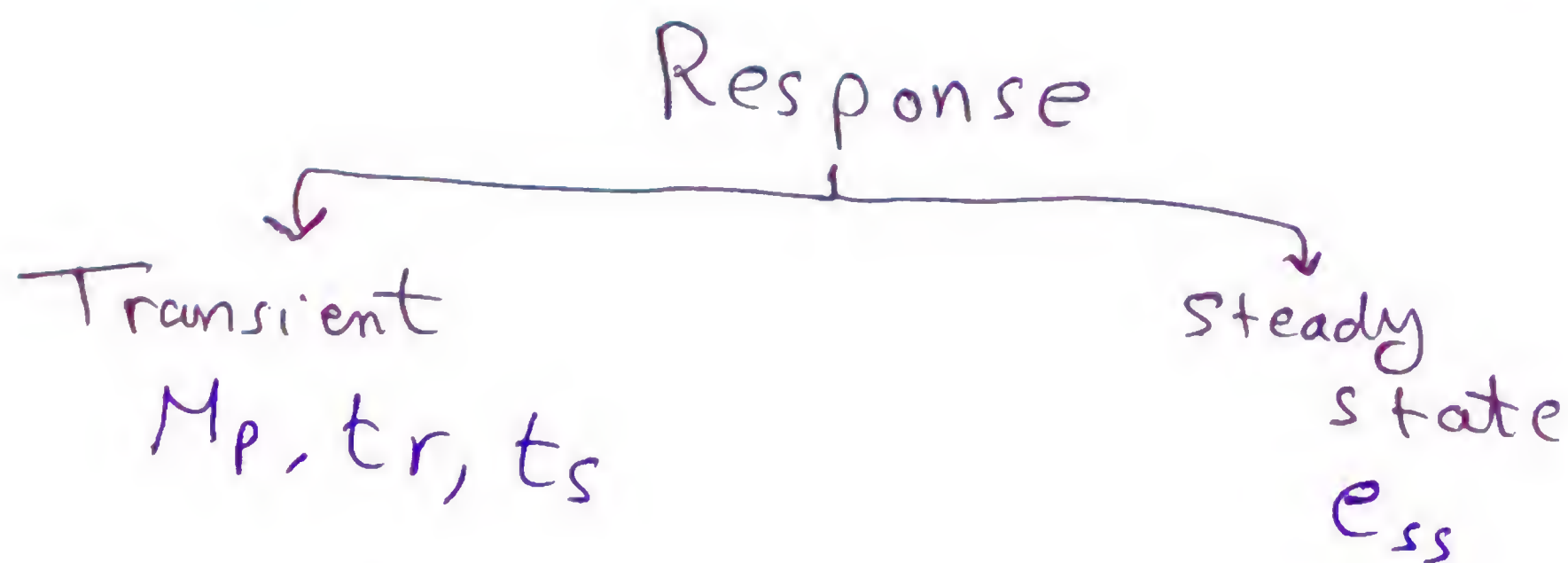
Add dynamic components to the system to change the overall system dynamics i.e. change the locations of C.L. poles.

Remember what we've done in practice :-

* Requirements

* Modelling of system dynamics (Analysis) \Rightarrow (Dynamics don't meet desired specs)

* Controller (Add Components)



* Transient Response Compensation

→ Lead Compensator

→ PD

* Steady State error Compensation

→ Lag Compensator

→ PI

* For overall compensation

→ Lead-Lag Compensator

→ PID

~~##~~ Lead Compensator :-

at $S_d: \sum \phi_z - \sum \phi_p \neq \pm 180$

desired

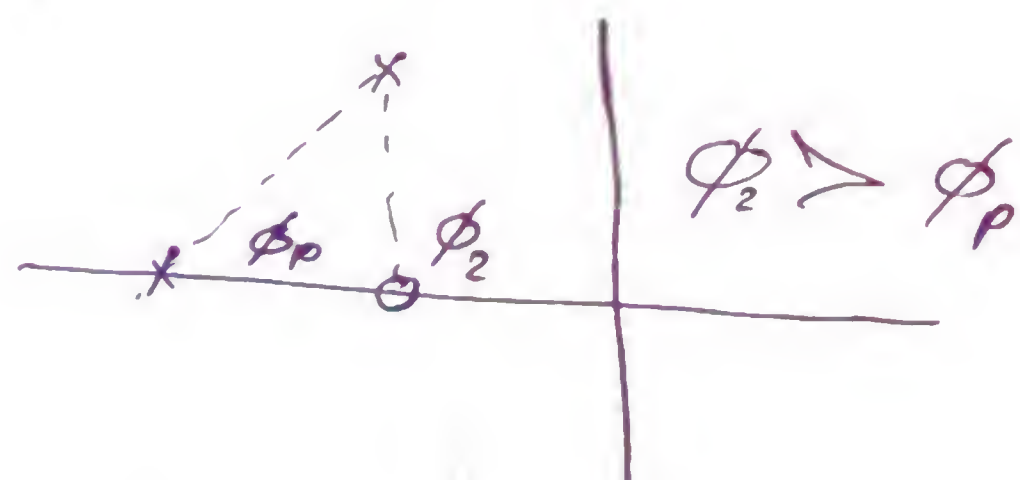
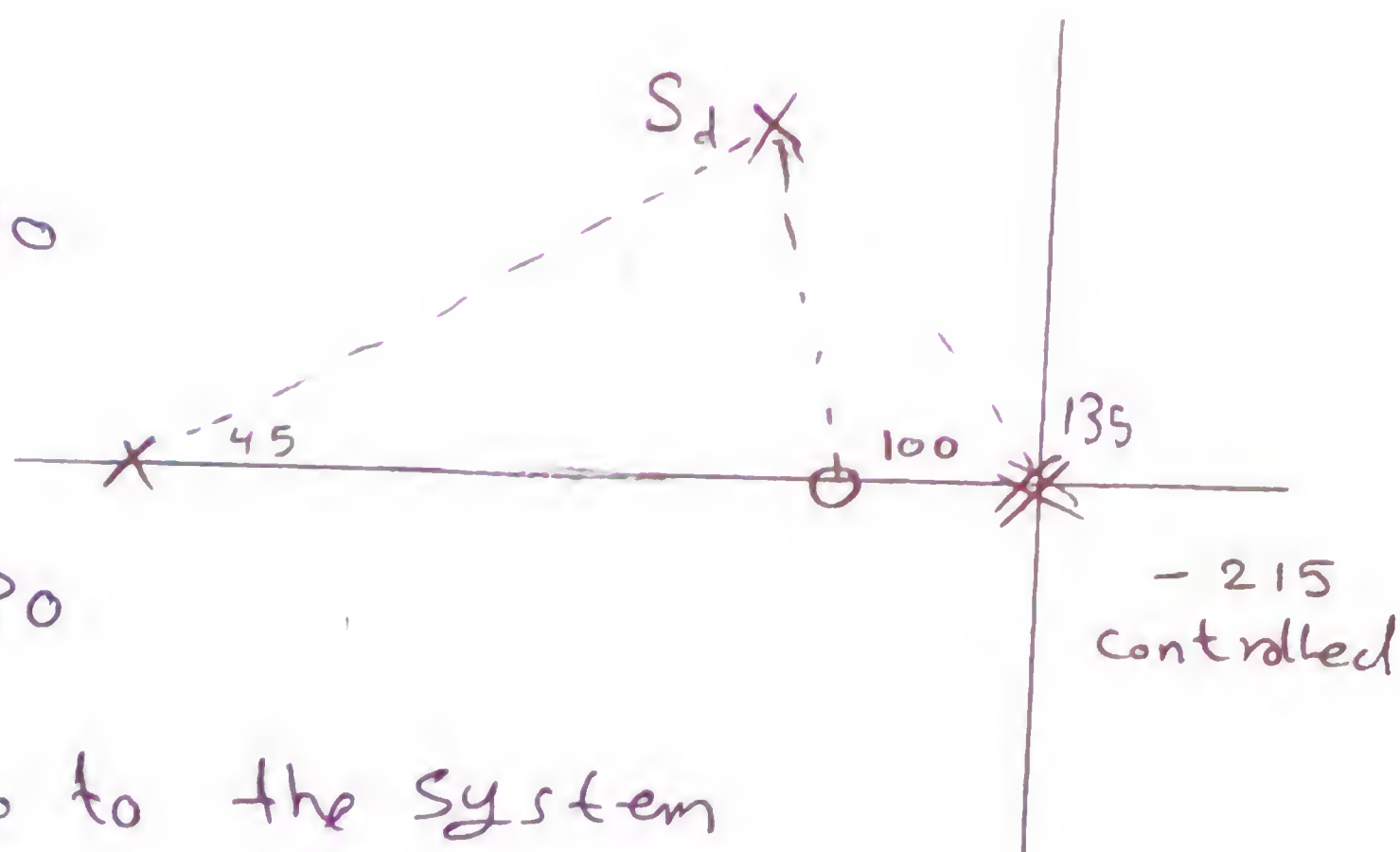
at $S_d: \sum \phi_z - \sum \phi_p = \pm 180$

By adding a zero and a pole to the system

$$\sum \phi_{zc} - \sum \phi_{pc} = -180 - \phi$$

$$\angle GH(s) + \phi_c = -180$$

Root locus Angle Condition



* Lag Compensator design

① Calculate $\beta = \frac{K_c}{K_{un}} \times 1.1 \rightarrow \text{Safety Factor}$

$K_{un} \rightarrow \text{uncontrolled}$

K_c : desired value of DC gain

K_{un} : the system value of DC gain

② Assume zero location (z_c) in the range of 10% from the 2nd dominant pole of the system:

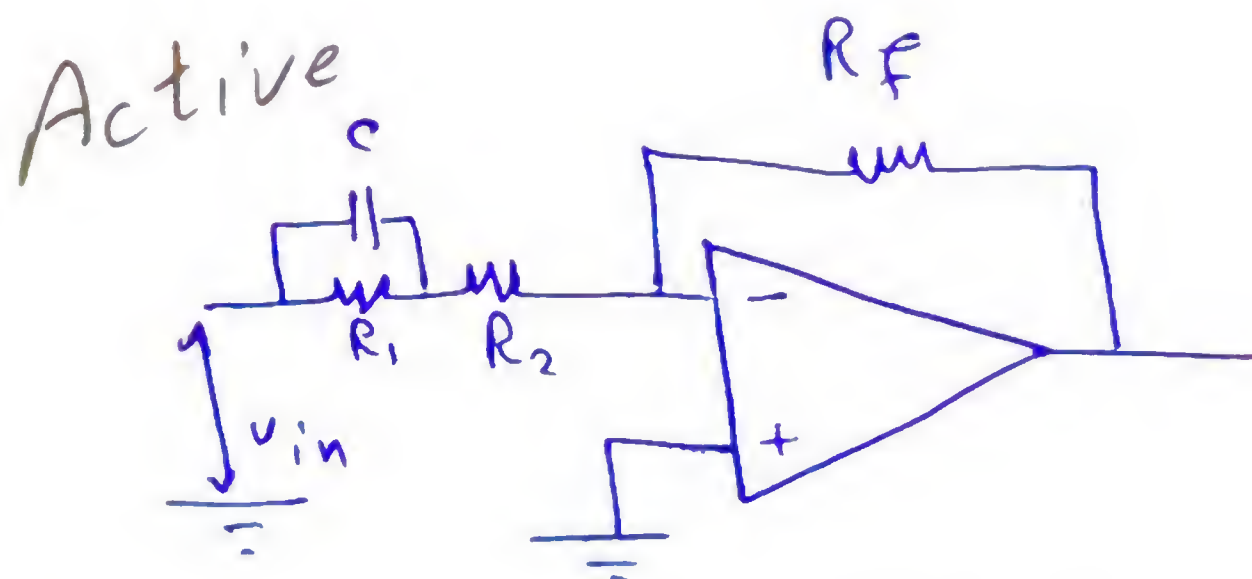
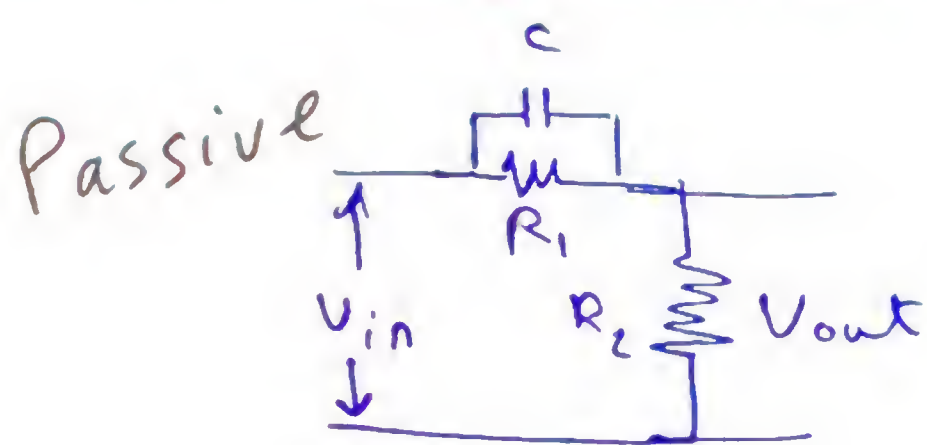
$$z_c = \frac{10}{100} p_2 \leftarrow \text{second dominant pole}$$

$$p_c = \frac{z_c}{\beta}$$

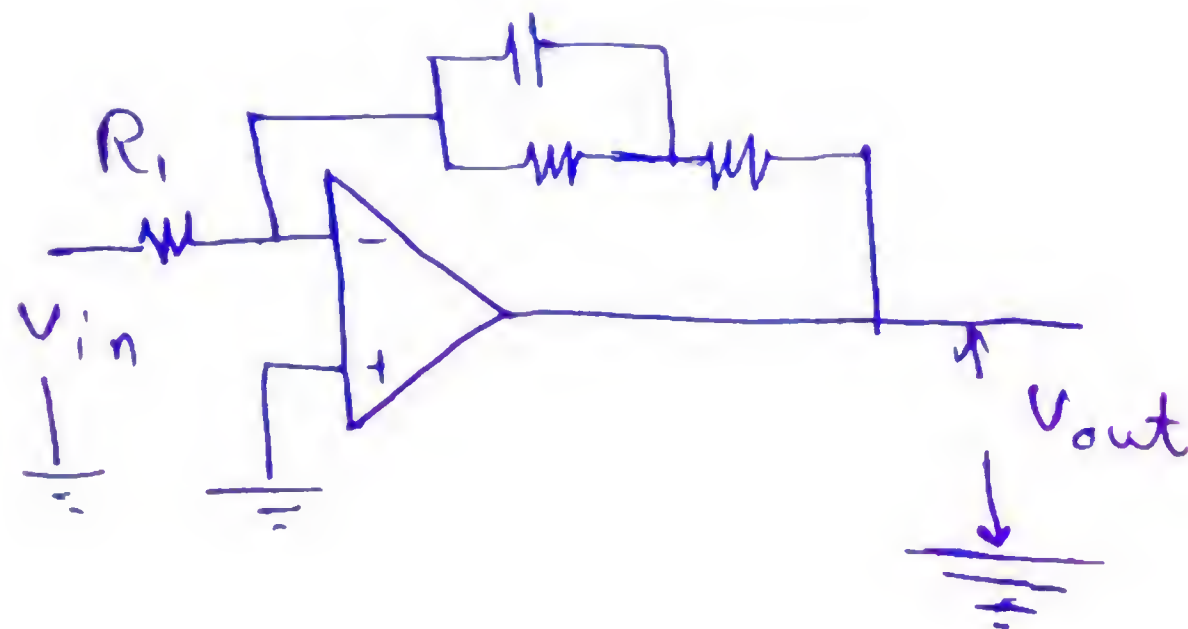
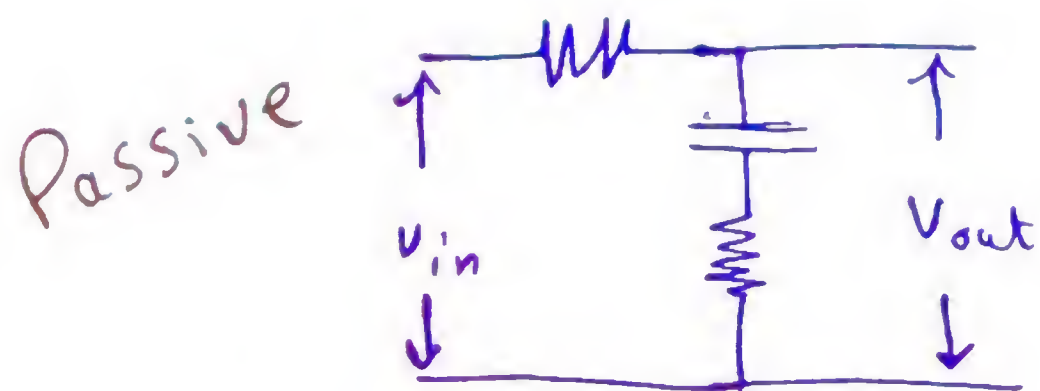
$$K_{dc} = \frac{z_c}{z_c / \beta} = \beta$$

$K_{dc} \leftarrow \text{of controller}$

* Lead Compensator



* Lag Compensator



Example :-

A type 1 system has an O.L.T.F $G_H(s) = \frac{K}{s(s+1)(s+4)}$
It is desired to compensate such system to match the following design specs:-

$$\zeta = 1/2$$

$$e_{ss} \leq 0.1$$

$$\omega_n = 2 \text{ rad/sec}$$

① desired locations for closed loop poles

$$s_{1,2} = -\zeta\omega_n \pm \omega_n \sqrt{1-\zeta^2}$$

$$= -1 \pm j\sqrt{3}$$

② from the angle condition

$$\angle G_H(s) + \phi_c = -180$$

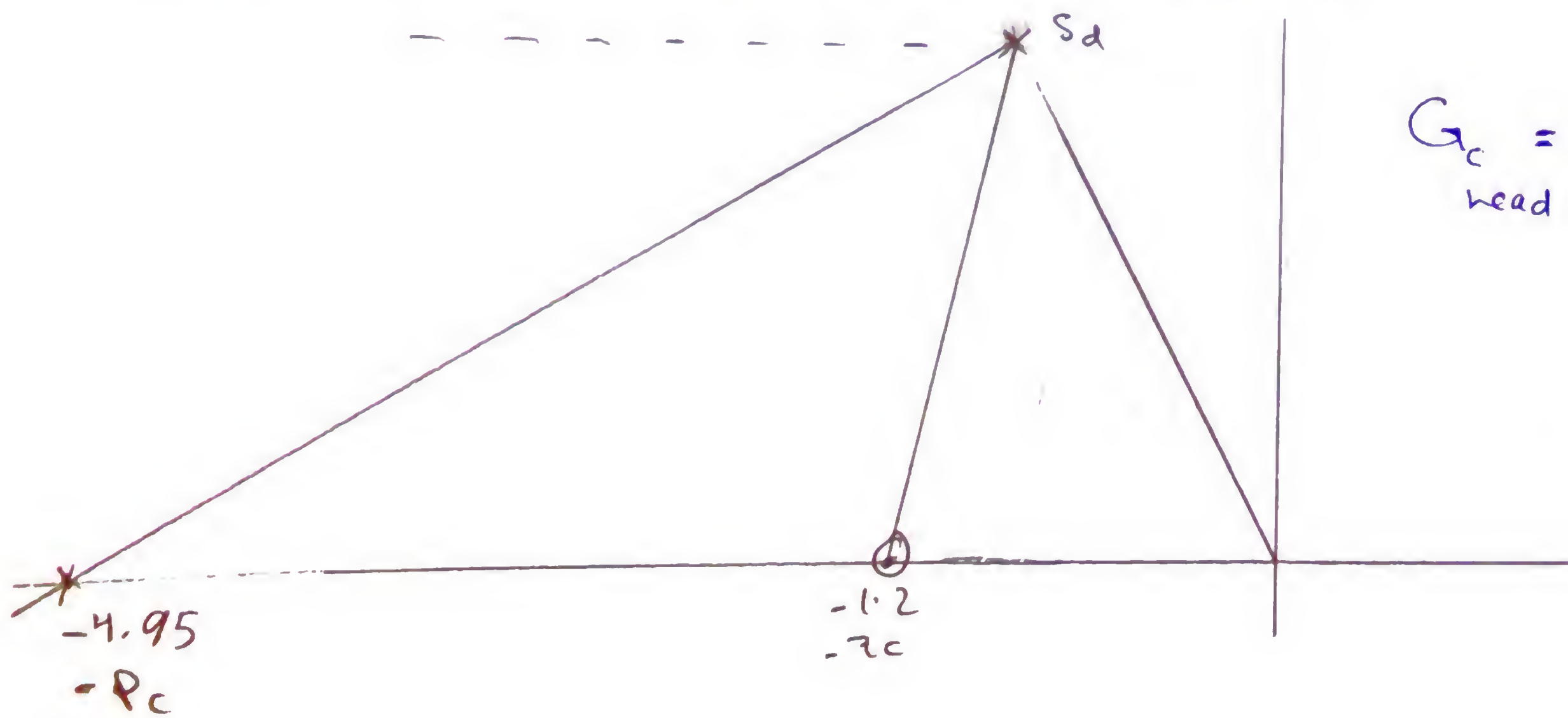
$$\angle \frac{K}{s(s+1)(s+4)} = 0 - \angle -1+j\sqrt{3} - \angle j\sqrt{3} - \angle 3+j\sqrt{3}$$

$$s = -1+j\sqrt{3} \quad + \phi_c = -180$$

$$-120 - 90 - 30 + \phi_c = -180$$

$$\Rightarrow \phi_c = 60$$

③ Determine the locations of P , Z_c & P_c



$$G_c = \frac{s+1.2}{s+4.95}$$

The total T.F.

$$GH(s) = \frac{K(s+1.2)}{s(s+1)(s+4)(s+4.95)}$$

④ Determine K from Magnitude condition

$$\|K GH(s)\| = 1$$

$$K = \frac{1}{\|GH(s)\|} = \frac{\|s\| \|s+1\| \|s+4\| \|s+4.95\|}{\|s+1.2\|} \bigg|_{s=-1+j\sqrt{3}} = 30$$

⑤ Check steady State error

$$\text{type 1 system} \Rightarrow e_{ss} = \frac{1}{K_v}$$

$$K_v = \lim_{s \rightarrow 0} s GH(s)$$

$$K_v = \lim_{s \rightarrow 0} \frac{s(s+1.2) \times 30}{s(s+1)(s+4)(s+4.95)} = 1.81$$

$$e_{ss} = \frac{1}{K_v} = 0.55$$

$$e_{ss} \leq 0.1$$

Required

to reduce e_{ss} Add Lag Compensator

$$\text{desired } e_{ss} = 0.1 \Rightarrow K_v = \frac{1}{e_{ss}} = 10$$

* Lag compensator design

$$\textcircled{1} K_{un} = 1.82$$

$$K_c = 10$$

$$\beta = \frac{10}{1.82} \times 1.1 = 6.04$$

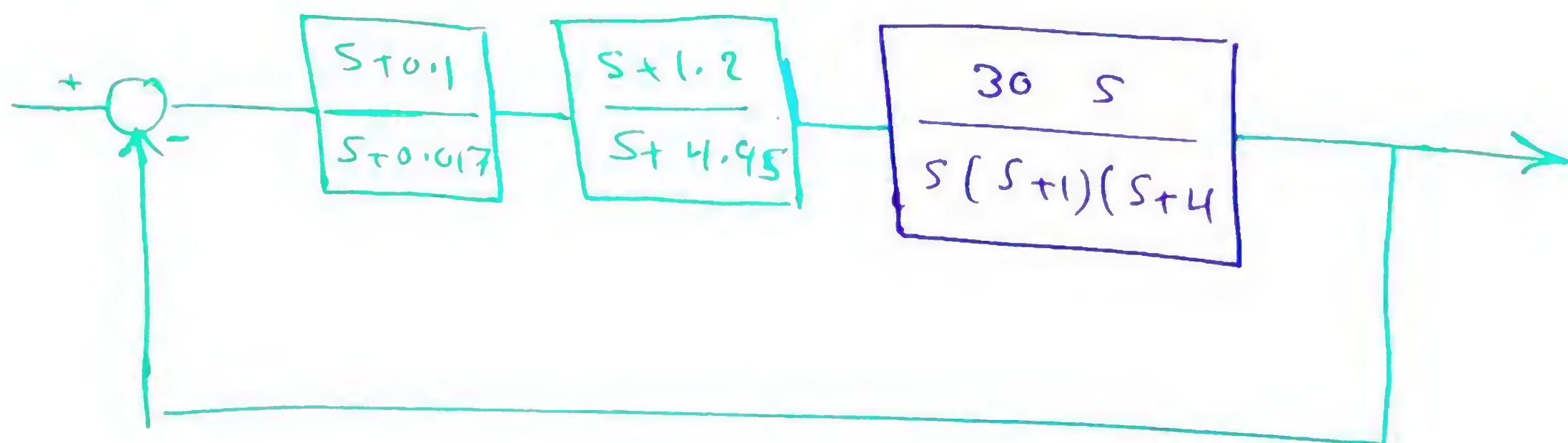
② Locations of z_c and p_c

$$z_c = \frac{10}{100} p_2 = 0.1 \times -1 = -0.1$$

$$p_c = \frac{z_c}{\beta} = \frac{-0.1}{6.04} = -0.017$$

$$G_c \text{ lag} = \frac{s + 0.1}{s + 0.017}$$

$$G(s) = \frac{30(s+0.1)(s+1.2)}{s(s+0.017)(s+1)(s+4)(s+4.95)}$$



Check e_{ss}

$$\lim_{s \rightarrow 0} s G(s) = \frac{30 \cancel{s} (s+0.1)(s+1.2)}{\cancel{s} (s+0.017)(s+1)(s+4)(s+4.95)} ; e_{ss} = 0.09$$